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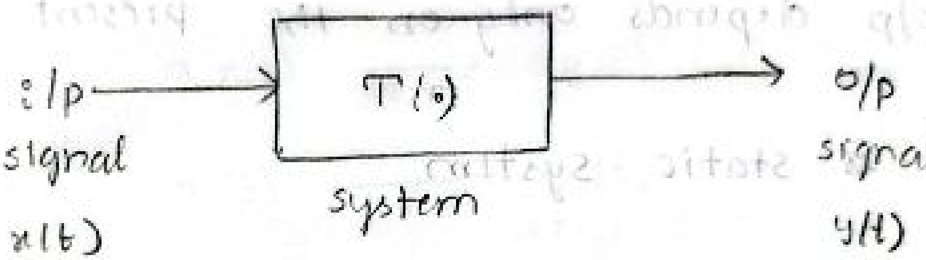
UNIT - III

SIGNAL TRANSMISSION THROUGH LINEAR SYSTEM

System :-

continuous time system is a device that operates on the continuous time ~~tputt~~ i/p signal to produce continuous time o/p signal



System Equation :-

$y(t) = T[x(t)]$
o/p ... i/p

i/p signal ⇒ Excitation

$x(t) \xrightarrow{T(\cdot)} y(t) = T[x(t)]$

o/p signal ⇒ Response

$x(t) \longrightarrow y(t) = T[x(t)]$

Types of systems

1) static Vs Dynamic system -

- static system is also known as memoryless system
- A sys is said to be static system if and if only the output of the system depends only on the present i/p of the system .
- Otherwise the sys is said to be Dynamic system

Q, check whether the following system is static (or) Dynamic system

i, $y(t) = a x(t) + b$

$t=0 \Rightarrow y(0) = a(x(0)) + b$

$1 \Rightarrow y(1) = a x(1) + b$

$-1 \Rightarrow y(-1) = a x(-1) + b$

$\therefore$ present o/p depends only on the present i/p.

$\therefore$ It is static system

ii, $y(t) = x(t+1)$

$t=0 \Rightarrow y(0) = x(1)$ [$\because$ present o/p depends on future i/p]

$\therefore$ It is dynamic system

iii, $y(t) = x(-t)$

$t=0 \Rightarrow y(0) = x(0)$

$t=1 \Rightarrow y(1) = x(-1)$ [$\because$ depends on past i/p]

$\therefore$ It is dynamic system

2, linear Vs Non linear systems -

system is said to be a linear system if and if only the system satisfies the -

i, Homogenity principle

ii, superposition principle

$$\text{i/p} \qquad\qquad \text{o/p}$$

$$x(t) \xrightarrow{T(\cdot)} y(t) = T[x(t)]$$

$$\left.a\,x(t) \longrightarrow a\,y(t)\right\} \text{Homogenity principle}$$

$$\left.\begin{array}{l} x_1(t) \longrightarrow y_1(t) \\ x_2(t) \longrightarrow y_2(t) \\ x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t) \end{array}\right\} \text{superposition principle}$$

Procedure to check linearity

$$\text{i/p} \qquad\qquad \text{o/p}$$

$$x(t) \xrightarrow{T(\cdot)} y(t) = T\,x(t)$$

1, $x_1(t) \longrightarrow y_1(t) = T\,x_1(t)$

2, $x_2(t) \longrightarrow y_2(t) = T\,x_2(t)$

3, $a_1x_1(t) + a_2x_2(t) \longrightarrow y_3(t) = T\,[a_1x_1(t) + a_2x_2(t)]$

4, $y_4(t) = a_1y_1(t) + a_2y_2(t)$

$\Rightarrow$ if $y_3(t) = y_4(t)$ then it is linear system

$$y_3(t) = y_4(t)$$

$$\boxed{T[a_1 x_1(t) + a_2 x_2(t)] = a_1 T(x_1(t)) + a_2 T(x_2(t))}$$

Q, Check the following system s are linear or not

i, $y(t) = \alpha x(t) + \beta$

1, $x_1(t) \rightarrow y_1(t) = \alpha x_1(t) + \beta$

2, $x_2(t) \longrightarrow y_2(t) = \alpha x_2(t) + \beta$

3, $a_1 x_1(t) + a_2 x_2(t) \longrightarrow y_3(t) = \alpha[a_1 x_1(t) + a_2 x_2(t)] + \beta$

4, $$y_4(t) = a_1 y_1(t) + a_2 y_2(t)$$

$$= a_1[\alpha x_1(t) + \beta] + a_2[\alpha x_2(t) + \beta]$$

$$y_3(t) \neq y_4(t)$$

$\therefore$ Non-linear

ii, $y(t) = x(t) + \dfrac{1}{x(t-1)}$

1, $x_1(t) \rightarrow y_1(t) = x_1(t) + \dfrac{1}{x_1(t-1)}$

2, $x_2(t) \rightarrow y_2(t) = x_2(t) + \dfrac{1}{x_2(t-1)}$

3, $a_1 x_1(t) + a_2 x_2(t) \rightarrow y_3(t) = a_1\left[x_1(t) + \dfrac{1}{x_1(t-1)}\right] + a_2\left[x_2(t) + \dfrac{1}{x_2(t-1)}\right]$

4, $y_4(t) = a_1 y_1(t) + a_2 y_2(t) = a_1\left[x_1(t) + \frac{1}{x_1(t-1)}\right] + a_2\left[x_2(t) + \frac{1}{x_2(t-1)}\right]$

$y_3(t) \neq y_4(t)$

Non linear system

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iii, $y(t) = x(t) + \frac{1}{x(t-1)}$

1, $x_1(t) \rightarrow y_1(t) = x_1(t) + \frac{1}{x_1(t-1)}$

2, $x_2(t) \rightarrow y_2(t) = x_2(t) + \frac{1}{x_2(t-1)}$

3, $a_1 x_1(t) + a_2 x_2(t) \longrightarrow y_3(t) = [a_1 x_1(t) + a_2 x_2(t)] + \frac{1}{a_1 x_1(t-1) + a_2 x_2(t-1)}$

4, $y_4(t) = a_1 y_1(t) + a_2 y_2(t) = a_1\left[x_1(t) + \frac{1}{x_1(t-1)}\right] + a_2\left[x_2(t) + \frac{1}{x_2(t-1)}\right]$

$y_3(t) \neq y_4(t)$

Non-linear system

iv, $\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 3y(t) = x(t)$

1, $x_1(t) \rightarrow y_1(t)$

$\frac{d^2 y_1(t)}{dt^2} + 2\cdot\frac{d\cdot y_1(t)}{dt} + 3y_1(t) = x_1(t)$ — ①

2, $x_2(t) \rightarrow y_2(t)$

$\frac{d^2 y_2(t)}{dt^2} + 2\cdot\frac{dy_2(t)}{dt} + 3y_2(t) = x_2(t)$ — ②

3, $a_1 x_1(t) + a_2 \cdot x_2(t) \longrightarrow a_1 \cdot y_1(t) + a_2 y_2(t)$

$$\frac{d^2}{dt^2}[a_1 y_1(t) + a_2 y_2(t)] + 2\frac{d}{dt}[a_1 \cdot y_1(t) + a_2 y_2(t)]$$

$$+3[a_1 y_1(t) + a_2 y_2(t)] = a_1 x_1(t) + a_2 x_2(t) \quad \text{—(3)}$$

4) linear combination of (1) & (2) –

$$= a_1\left[\frac{d^2 y_1(t)}{dt^2} + 2\cdot\frac{d}{dt} y_1(t) + 3y_1(t)\right] + a_2\left[\frac{d^2}{dt^2} y_2(t) + 2\frac{d}{dt}y_2(t) + 3\cdot y_2(t)\right]$$

$$= a_1 x_1(t) + a_2 \cdot x_2(t) \quad \text{—(4)}$$

(3) = (4)

linear system

v) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

1) $x_1(t) \rightarrow y_1(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau$

2) $x_2(t) \longrightarrow y_2(t) = \int_{-\infty}^{t} x_2(\tau) d\tau$

3) $a_1 x_1(t) + a_2 x_2(t) \longrightarrow a_1 y_1(t) + a_2 y_2(t)$

$$a_1 y_1(t) + a_2 y_2(t) = \int_{-\infty}^{t} [a_1 x_1(t) + a_2 x_2(t)]\, d\tau$$

4) $a_1 y_1(t) + a_2 \cdot y_2(t) = a_1 \int_{-\infty}^{t} x_1(\tau) \cdot d\tau + a_2 \int_{-\infty}^{\tau} x_2(\tau) \cdot dt$

3 = 4

linear system

3, Time Invariant Vs Time varying systems

→ If the i/p & o/p characteristics of the system does not change w.r.t time such kind of system is said to be Time Invariant system

→ otherwise Time varying system.

i/p $\qquad$ o/p

$x(t) \longrightarrow y(t) = T[x(t)]$

1, Delay i/p

$x(t-t_0) \longrightarrow y(t, t_0) = T[x(t-t_0)]$

2, Delay o/p $\quad y(t-t_0)$

if $(y(t,t_0)) = y(t-t_0)$

system is TIV

Q, check whether the following system is time invariant (or) time varying system

i, $y(t) = ax(t) + b$

1, Delay i/p $\Rightarrow x(t-t_0) \rightarrow y(t,t_0)$

$y(t,t_0) = a \cdot x(t-t_0) + b \quad$ —①

2, delay o/p $\Rightarrow$ $y(t-t_0) = a \cdot x(t-t_0) + b$ — ②

$$y(t, t_0) = y(t-t_0)$$

Time Invariant

iii, $y(t) = x(-t)$

i/p $\rightarrow$ o/p
$x(t) \rightarrow y(t)$

1, Delay i/p

$$y(t, t_0) = x(-t - t_0)$$

2, Delay o/p

$$y(t-t_0) = x(-(t-t_0))$$

$$= x(-t+t_0)$$

$$y(t, t_0) \neq y(t-t_0)$$

TV

iii, $y(t) = x(t/2)$

4) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

IMP

1) Delay I/p

$$x(t-t_0) \longrightarrow y(t, t_0)$$

$$y(t,t_0) = \int_{-\infty}^{t} x(\tau - t_0)\, d\tau \quad -①$$

2) delay o/p

$$y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau \quad -②$$

In eqn ① $\tau - t_0 = t_1$

$d\tau = dt_1$

$$\tau = -\infty \Rightarrow t_1 = -\infty$$

$$\tau = t \Rightarrow t_1 = t - t_0$$

$$y(t,t_0) = \int_{-\infty}^{t-t_0} x(t_1)\, dt_1$$

$$t_1 \to t$$

$$y(t,t_0) = \int^{t-t_0} x(t)\, dt$$

$$y(t,t_0) = y(t-t_0)$$

TIV

Note :-

If the coeff of diff equ are const. then the system is said to be TIV.

5, $\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 3\,y(t) = 1x(t)$ ..

coefficients are constants. Hence the system is time-invarient system.

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4, Causal Vs Non-causal systems:-

The system is said to be causal system if & it only the o/p of the system depends on present i/p, Past i/p & past o/p only else it is non-causal system.

Eg:-

check wheather the following system is c (or) n-c system

i, $y(t) = e^{x(t)}$

if $t=0$, $y(0) = e^{x(0)}$

if $t=-1$, $y(-1) = e^{x(-1)}$

if $t=1$, $y(1) = e^{x(1)}$

Present o/p depends only on present i/p.

∴ The system is causal system

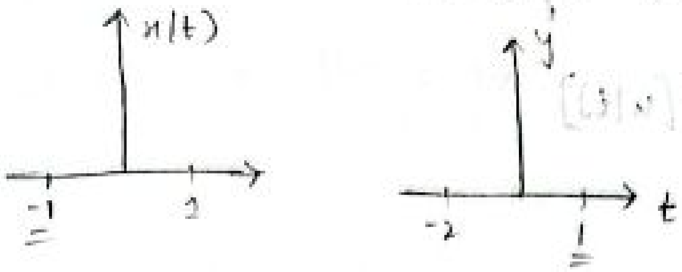
ii, $y(t) = x(-t)$

if $t=0$, $y(0) = x(0)$ ⇒ present i/p = present o/p

if $t=-1$, $y(-1) = x(1)$ ⇒ past i/p ≠ present o/p

if $t=1$, $y(1) = x(-1)$ ⇒ present i/p ≠ past i/p

$\therefore$ non-causal system



iii, $y(t) = y(t-1) + x(t) + x(t-1)$

if $t=0 \Rightarrow y(0) = y(-1) + x(0) + x(-1)$

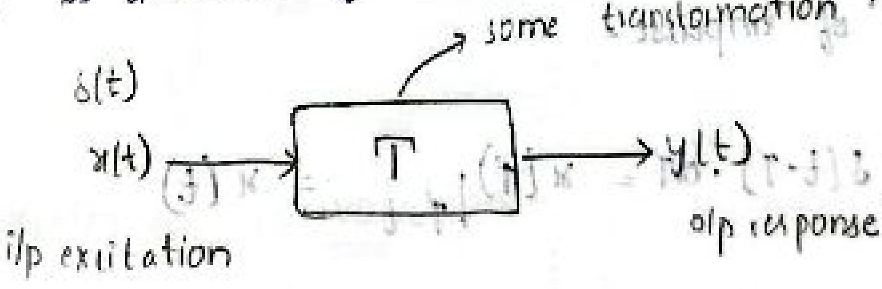
, $t=1$ $\quad y(1) = y(0) + x(1) + x(0)$

$t=-1$ $\quad y(-1) = y(-2) + x(-1) + x(-2)$

$\therefore$ causal system

Impulse Response of the system

- when i/p to the system is an impulse signal, the corresponding response of system is impulse response of the system
- It is denoted by $h(t)$.



$y(t) = T[x(t)]$

$h(t) = T[\delta(t)]$

Step Response -

- when i/p to the system is unit step signal the corresponding response of the system is unit step response of the system

It is denoted by 's(t)'

$$s(t) = T[u(t)]$$

CONVOLUTION -

$$\underset{x(t)}{\overset{i/p}{}} \longrightarrow \boxed{T(\cdot)} \longrightarrow \overset{o/p}{y(t) = T[x(t)]}$$

LTI system

i/p		o/p
$x(t)$	$\longrightarrow$	$y(t) = T[x(t)]$
$\delta(t)$	$\longrightarrow$	$h(t) = T[\delta(t)]$
$\delta(t-\tau)$	$\xrightarrow{T.I}$	$h(t-\tau) = T[\delta(t-\tau)]$
$x(t) \cdot \delta(t-\tau)$	$\xrightarrow{\text{homogeneous}}$	$x(\tau) \cdot h(t-\tau)$
$\int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) \cdot d\tau$	$\xrightarrow[\text{principle}]{\text{superposition}}$	$\int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$

sampling prop. of impulse -

$$\int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) \cdot d\tau = x(\tau)\big|_{\tau=t} = x(t)$$

i/p		o/p
$x(t) = \int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) \cdot d\tau$	$\longrightarrow$	$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$

- the LTI system is always characterized by using its impulse response i.e $h(t)$.

- The o/p of the LTI system is always the linear convolution b/w i/p & impulse response of the system

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$$

LTI

Linear convolution of 2 signals -

$$y(t) = x(t) * h(t)$$

$$y(\tau) = \int_{-\infty}^{\infty} x(\tau) \cdot h(\tau) \, d\tau$$

$$y(\tau) = \int_{-\infty}^{\infty} x(\tau) \cdot h(-\tau) \cdot d\tau \rightarrow S1 - \text{folding}$$

$$y(\tau) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau \rightarrow [S_2 - \underline{R.S}]$$

Right shift

Properties of convolution -

1) $x(t) * h(t) = h(t) * x(t)$ [commutative]

2) $(x(t) * h_1(t)) * h_2(t) = x(t) * (h_1(t) * h_2(t))$ [Associative]

3) $x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$

[Distributive]

4) shifting property-

$$x(t) * h(t) = y(t)$$

$$x(t-t_1) * h(t-t_2) = y(t-t_1-t_2) \quad R.S$$

$$x(t+t_1) * h(t+t_2) = y(t+t_1+t_2) \quad L.S$$

5) scaling property-

If $x(t) * h(t) = y(t)$ then

$$\boxed{x(at) * h(at) = \frac{1}{|a|} y(at)}$$

6) differentiation property-

$$\frac{d}{dt}[x(t) * h(t)] = x(t) * \frac{d}{dt} h(t) = \frac{d}{dt} x(t) * h(t)$$

7) Integration property-

$$\int_{-\infty}^{t} [x(t) * h(t)] \cdot dt = \left[\int_{-\infty}^{t} x(t) \cdot dt\right] * h(t)$$

$$= x(t) * \int_{-\infty}^{t} h(t) \cdot dt$$

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8) convolution theorem :-

$$x(t) * h(t) \xleftrightarrow{FT} X(\omega) \cdot H(\omega)$$

$$x(t) * h(t) = IFT\,[X(\omega) \cdot H(\omega)]$$

9) Multiplication theorem :-

$$x(t) \cdot h(t) \xleftrightarrow{FT} \frac{1}{2\pi}\,[X(\omega) * H(\omega)]$$

$$X(\omega) * H(\omega) = 2\pi\ FT\,[x(t) \cdot h(t)]$$

- The linear convolution can be used to compute the o/p of LTI system.
- The o/p of LTI system is always the linear convolution b/w i/p & Impulse response of system.
- The linear convolution b/w $x(t)$ & $h(t)$ can be computed using 2 different methods.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = IFT\,[X(\omega)\,H(\omega)]$$

⇓ Time domain method (or) graphical method

⇓ Frequency domain method.

10) $\boxed{x(t) * \delta(t) = x(t)}$

① T.D Method

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = x(\tau)\Big|_{\tau=t} = x(t)$$

sampling property of impulse

2, F.D method

$$x(t) * \delta(t) = IFT[X(\omega) \cdot 1] = x(t)$$

$\Rightarrow$ Any signal convolution with impulse signal results in same signal

Eg:- $x(t) * \delta(t - t_0)$

i, $x(t) * \delta(t) = x(t)$

shifting prop of convolution

If $x(t) * h(t) = \delta(t)$ then $x(t-t_1) * h(t-t_2) = y(t-t_1-t_2)$

ii, $x(t-t_1) * \delta(t-t_2)$

~~Q, find~~ $x(t) * \delta(t) = x(t)$

$x(t-t_1) * \delta(t-t_2) = x(t-t_1-t_2)$

If $t_1 = 0$, $t_2 = t_0$

$x(t) * \delta(t-t_0) = x(t-t_0)$

Q, Find the linear convolution b/w the two following signals (using graphical method)

i, $x(t) = e^{-\alpha t} \cdot u(t)$

$h(t) = u(t)$

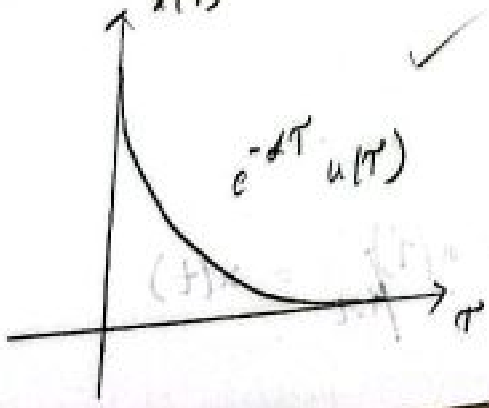
l_2 = l.l of 2

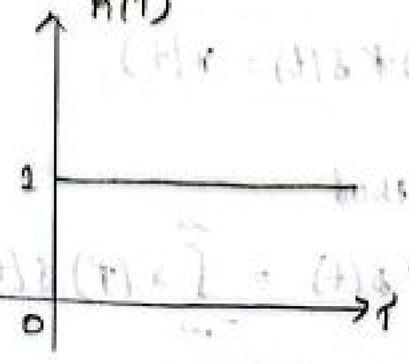
u_2 = u.l of 2

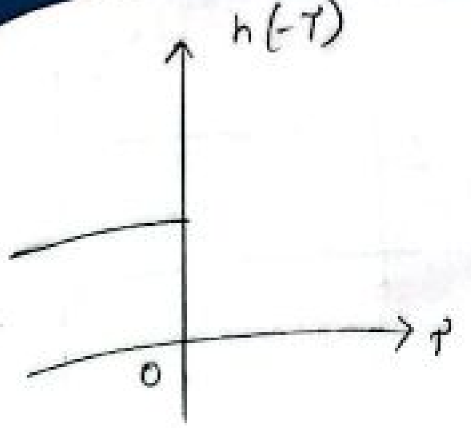
l_1 = lower limit of sig 1

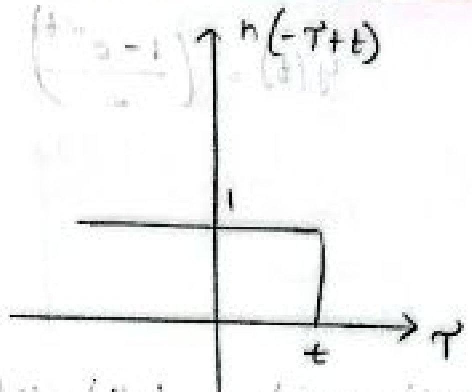
u_1 = upper limit of sig 1

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$









$l_1 = 0 \quad u_1 = \infty$

$l_2 = 0 \quad u_2 = \infty$

$t \Longrightarrow 0$ to ∞

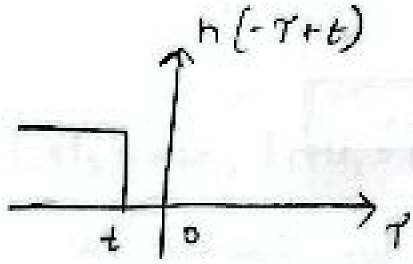
$l_1 + l_2 = 0$

$l_1 + u_2 = \infty$

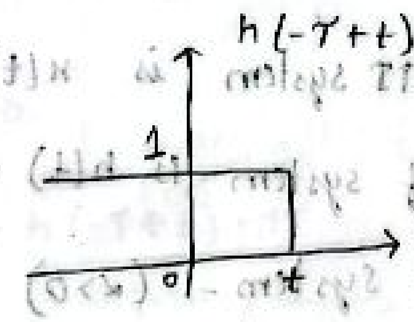
$l_2 + u_1 = \infty$

$u_1 + u_2 = \infty$

for $t < 0$



for $0 < t < \infty$



b'coz of $h(-\tau+t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{0} 0 \;+\; \int_{0}^{t} e^{-\alpha\tau} u(t)\, d\tau + \int_{t}^{\infty} 0$$

b'coz of $x(t)$ it exists from 0

in 0 to t range $u(t)=1$

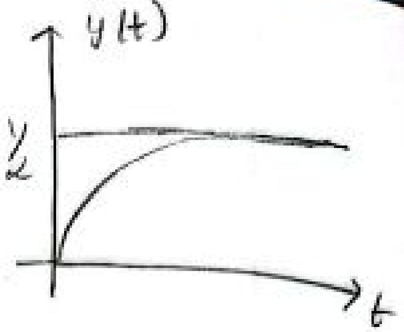
$h(-\tau+t)$ it doesn't exist in this region

$$= \int_{0}^{t} e^{-\alpha\tau}\, d\tau$$

$$= \left[\frac{e^{-\alpha\tau}}{-\alpha}\right]_0^t = -\frac{1}{\alpha}\left[e^{-\alpha t} - 1\right] = \left[\frac{1 - e^{-\alpha t}}{\alpha}\right]$$

for $0 < t < \infty$

$$y(t) = \left(\frac{1-e^{\alpha t}}{\alpha}\right) \cdot u(t)$$



ii, $x(t) = u(t)$, $h(t) = u(t)$

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Q, The i/p to the LTI system is $x(t) = e^{-\alpha t} u(t)$ & the impulse response of system is $h(t) = e^{\alpha t} u(-t)$. find the o/p of the system - $(\alpha > 0)$

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t) = h(t) * x(t)$

i/p LTI system o/p

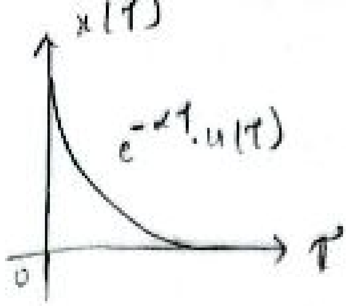
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

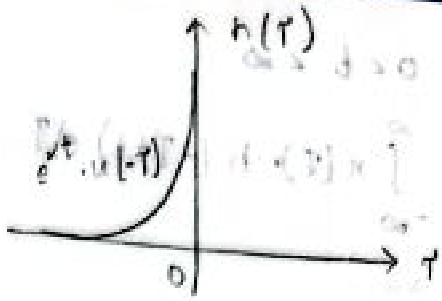
for $-\infty < t < \infty$

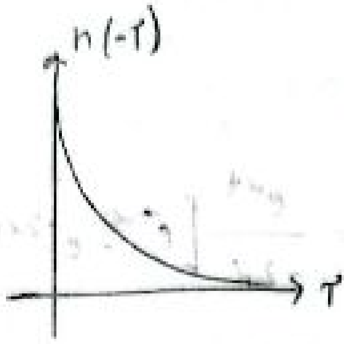
$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau) \cdot d\tau$$

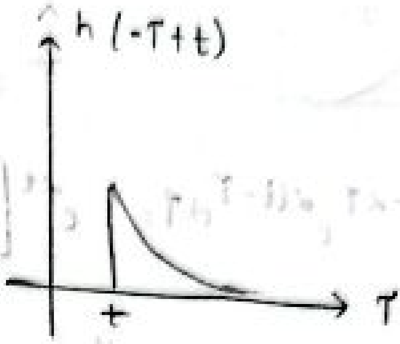
$$x(\tau) = e^{-\alpha t} u(\tau)$$

$$h(\tau) = e^{\alpha t} u(-\tau)$$



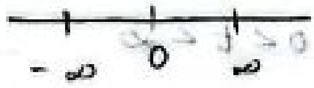






$l_1=0,\ u_1=\infty$

$l_2=-\infty,\ u_2=0$ $\qquad l_1+l_2=-\infty,\ l_1+u_2=0,\ l_2+u_1=0,\ u_1+u_2=\infty$

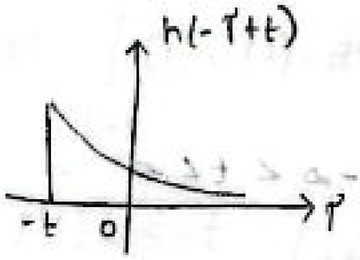


Case (i) :- $-\infty<t<0$

$$y(t)=\int_{-\infty}^{\infty}x(\tau)\cdot \underline{h(-\tau+t)}\cdot d\tau \quad \rightarrow RS$$

$h(-\tau+t)\rightarrow LS$

$t=$ -ve values $\rightarrow LS$



for $-\infty<t<0$

$$y(t)=\int_{0}^{\infty}e^{-\alpha\tau}\cdot e^{\alpha(t-\tau)}\cdot d\tau$$

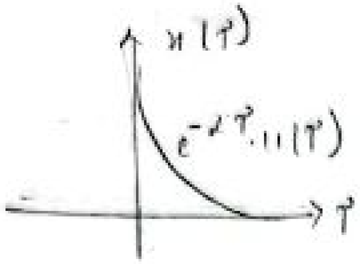
$$=\int_{0}^{\infty}e^{-\alpha\tau}\cdot e^{\alpha t}\cdot e^{-\alpha\tau}\cdot d\tau = e^{\alpha t}\int_{0}^{\infty}e^{-2\alpha\tau}\cdot d\tau$$

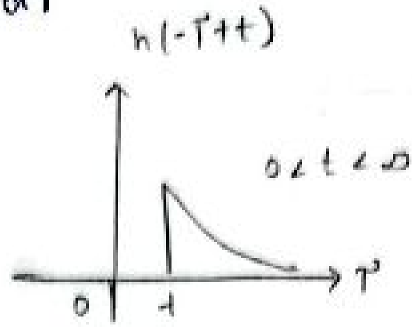
$$=e^{\alpha t}\left(\frac{e^{-2\alpha\tau}}{-2\alpha}\right)_{0}^{\infty}$$

$$\boxed{y(t)=\frac{1}{2\alpha}\cdot e^{\alpha t}}$$

Case ii :- $0 < t < \infty$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\cdot h(-\tau+t)\, d\tau$$





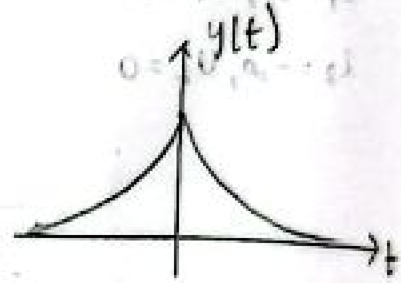
$$y(t) = \int_{0}^{\infty} e^{-\alpha\tau} e^{\alpha(t-\tau)} d\tau = e^{\alpha t}\left[\frac{e^{-2\alpha\tau}}{-2\alpha}\right]_{t}^{\infty} = \frac{e^{\alpha t}}{-2\alpha}\left[e^{-\infty} - e^{-2\alpha t}\right]$$

$0 \rightarrow t = 0$

$t \rightarrow t = \infty$

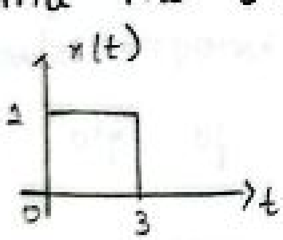
$$\boxed{y(t) = \frac{e^{-\alpha t}}{2\alpha}}$$

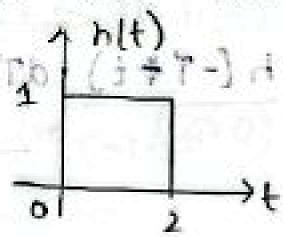
$$y(t) = \begin{cases} \frac{1}{2\alpha} \cdot e^{\alpha t} & \text{for } -\infty < t < 0 \\ \frac{1}{2\alpha}\, e^{-\alpha t} & \text{for } 0 < t < \infty \end{cases}$$



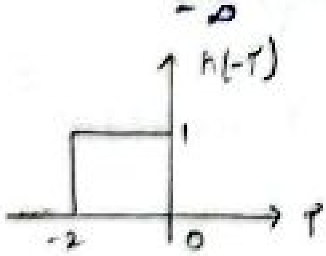
Q, Find the L.C of the two following signals -

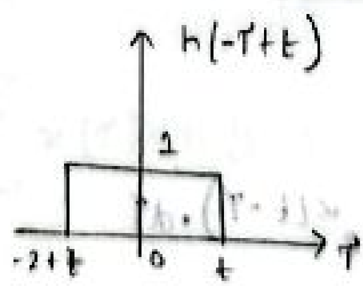
i,





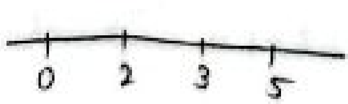
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\, d\tau \quad \text{for } -\infty < t < \infty$$



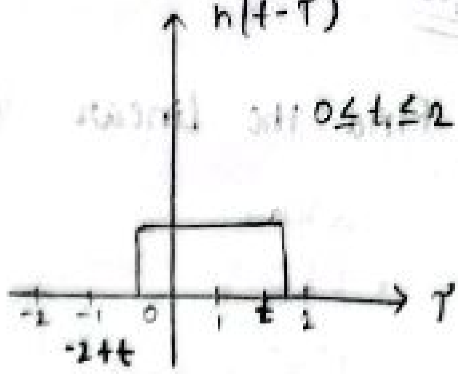


$l_1 = 0$, $l_2 = 0$

$u_1 = 3$, $u_2 = 2$ $\qquad l_1 + l_2 = 0$, $l_1 + u_1 = 3$, $l_1 + u_2 = 2$, $u_1 + u_2 = 5$



case i :- $0 \le t \le 2$



$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau$$

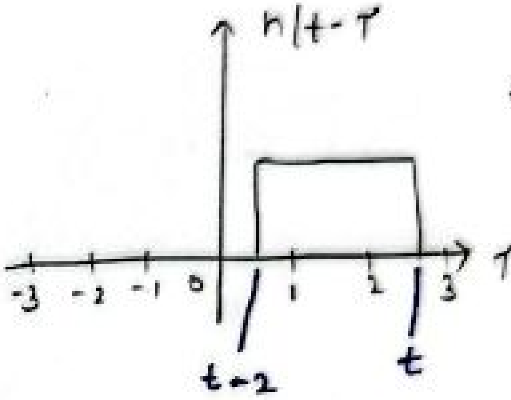
$$y(t) = \int_{0}^{t} (1)(1)\, d\tau$$

$$\boxed{y(t) = t}$$

$t = 1.999$

$h(-\tau + 1.999)$

case ii :- $2 \le t \le 3$

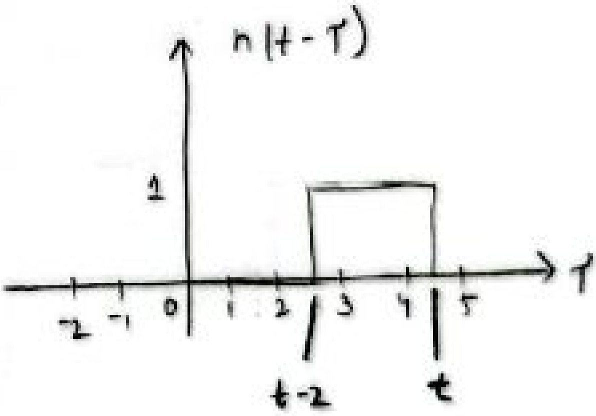


$t = 2.999$

$$y(t) = \int_{t-2}^{t} (1)\cdot(1)\, d\tau$$

$$\boxed{y(t) = 2}$$

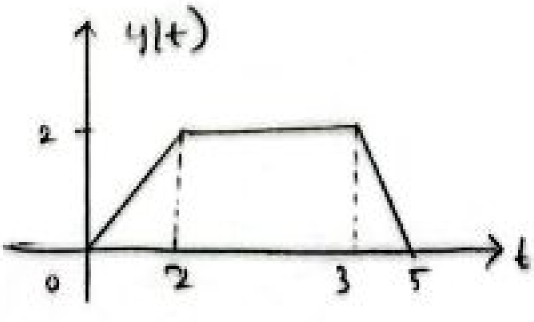
case iii :- $3 \le t \le 5$



$$y(t) = \int_{t-2}^{3} (1)\cdot(1)\cdot d\tau$$

$$y(t) = 5 - t$$

$$y(t) = \begin{cases} t & \text{for } 0 \le t \le 2 \\ 2 & \text{for } 2 \le t \le 3 \\ 5-t & \text{for } 3 \le t \le 5 \end{cases}$$



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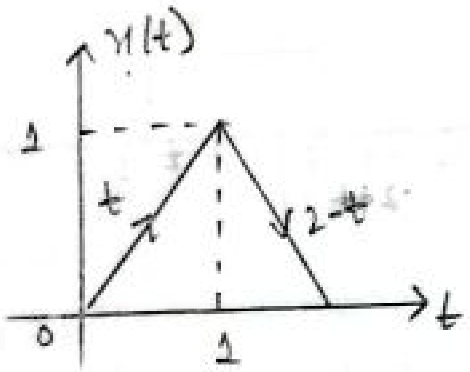
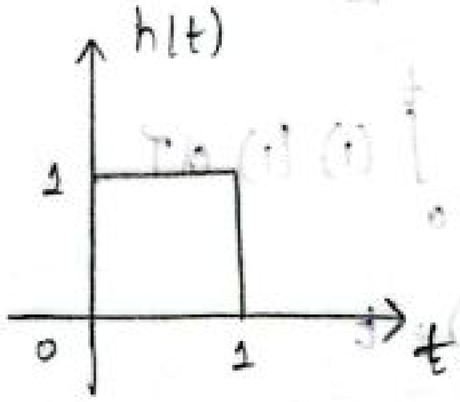
Q, Find the linear convolution b/w 2 signals



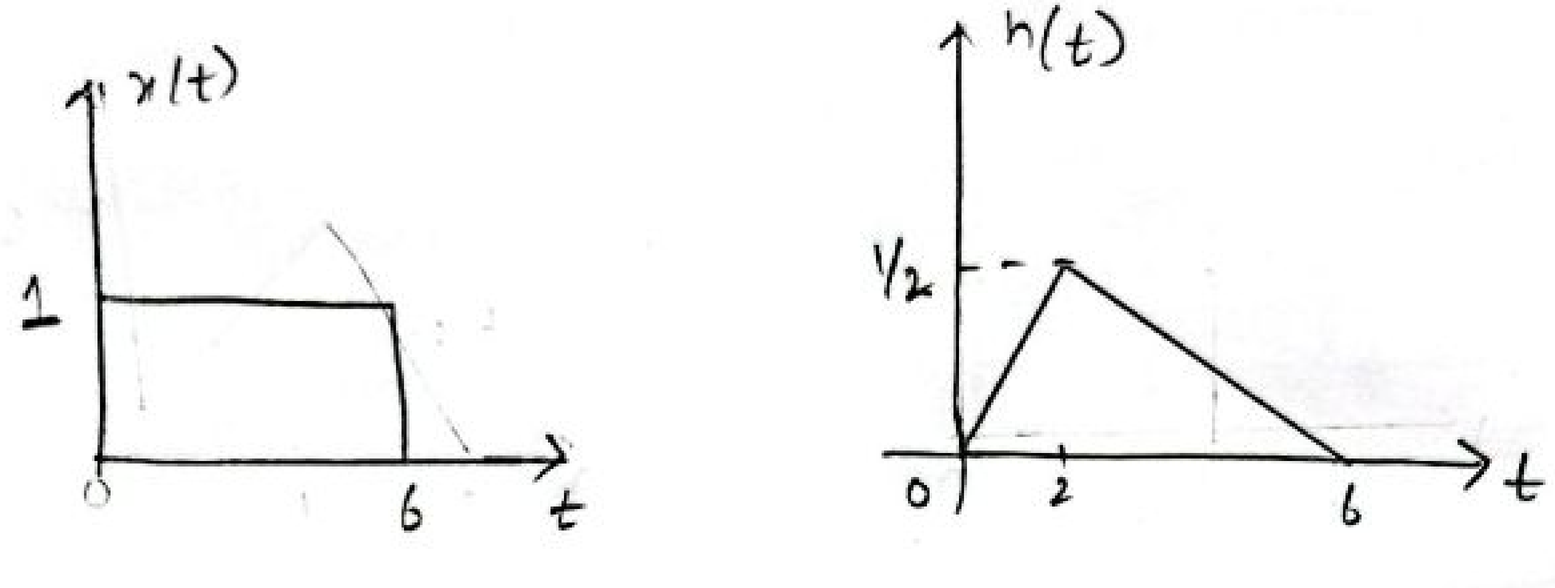


$0 \le t \le 1$

$1 \le t \le 2$

$2 \le t \le 3$

Q, Find the linear convolution b/w given 2 signals.



$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

0 2 6 8 12

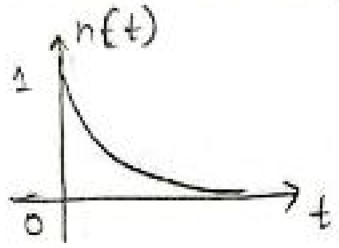
$0 \le t \le 2$

Causal Vs Non-Causal system -

The system is said to be causal if & only if the impulse response of the system $h(t)=0$ for $t<0$

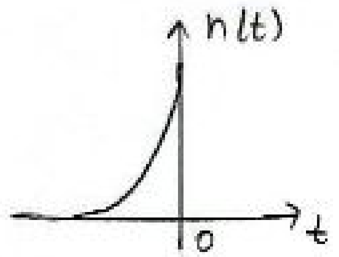
Eg:- 1) The impulse response of the system is $h(t)=e^{-3t}u(t)$

check whether C (or) N.C



$\therefore$ casual

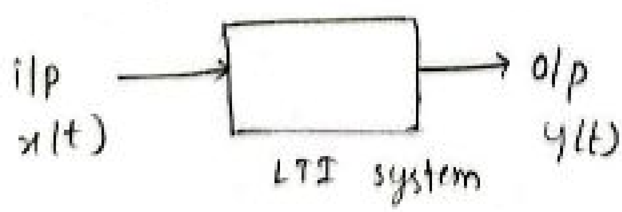
2) $h(t)=e^{2t}\ u(-t)$



stable Vs unstable System -

BIBO stable system -

The system is stable if & it only for every bounded i/p the system produces bounded o/p, then only it is stable system



BI

$|x(t)| \leq M_x < \infty \quad \forall t$

BO

$$|y(t)| \le M_y < \infty \quad \forall t$$

Q) check wheather the system is stable (or) unstable -

$$y(t) = \int_{-\infty}^{t} x(\tau)\cdot d\tau$$

BI

$$x(t) = u(t)$$

$$y(t) = \int_{-\infty}^{t} u(\tau)\cdot d\tau$$

$$= \int_{0}^{t} u(\tau)\cdot d\tau = \int_{0}^{t} 1\cdot d\tau = t$$

$$y(t) = t \longrightarrow \text{unbounded system}$$

<u>M-②</u> :-

<u>stability</u> <u>based</u> <u>on</u> <u>the</u> <u>impulse</u> <u>response</u> :-

for BIBO stable system,

BI :- $|x(t)| \le M_x < \infty$

BO :- $|y(t)| \le M_y < \infty$

$$\underset{x(t)}{i/p} \longrightarrow \underset{LTI}{\boxed{h(t)}} \longrightarrow \underset{y(t)}{o/p}$$

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$|y(t)| = \left| \int_{-\infty}^{\infty} h(\tau)\cdot x(t-\tau)\, d\tau \right|$$

$$|z_1 + z_2 + z_3| \le |z_1| + |z_2| + |z_3|$$

$$\left| \sum_{i=1}^{\infty} z_i \right| \le \sum_{i=1}^{\infty} |z_i|$$

$$|y(t)| \leq \int_{-\infty}^{\infty} h(\tau)\cdot x(t-\tau)\,d\tau$$

$$|z_1 z_2| \leq |z_1|\cdot|z_2|$$

$BI \Rightarrow |x(t)| \leq M_x$

$ly \; |x(t-\tau)| \leq M_x$

$$|y(t)| \leq M_x \int_{-\infty}^{\infty} |h(\tau)|\cdot d\tau$$

$BO \Rightarrow |y(t)| \leq M_y < \infty$

This is possible if $\int_{-\infty}^{\infty} |h(\tau)|\cdot d\tau < \infty$

If $\int_{-\infty}^{\infty} |h(t)|\cdot dt < \infty$ then the system is stable

*

Q, check whether the following system is S or US -

i, $h(t) = e^{-3t}\,u(t)$

$$\int_{0}^{\infty} e^{-3t}\,u(t) = \left[\frac{e^{-3t}}{-3}\right]_0^{\infty}$$

$$= \frac{1}{3}$$

ii, $h(t) = e^{3t}\,u(t)$

$$\int_{0}^{\infty} e^{3t}\cdot dt = \left[\frac{e^{3t}}{3}\right]_0^{\infty}$$

$$= -\frac{1}{3}$$

unstable

• If the i/p to the LTI system is an unit-step signal, the corresponding response of the system is known as step response of the system

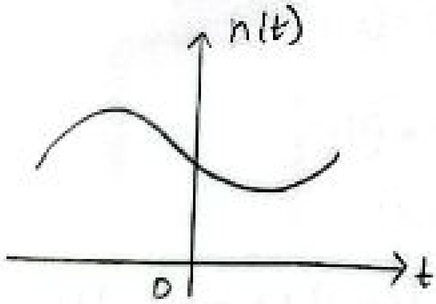
LTI

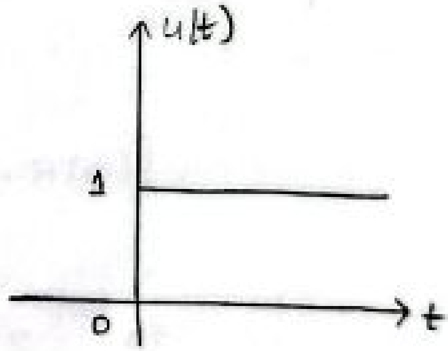
$$y(t) = x(t) * h(t)$$

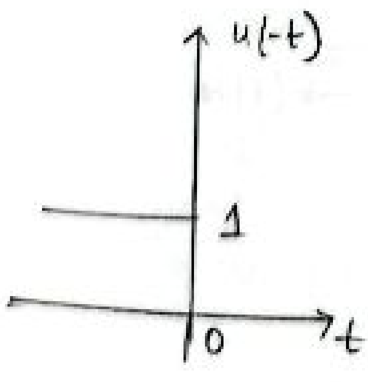
$$s(t) = u(t) * h(t) = h(t) * u(t)$$

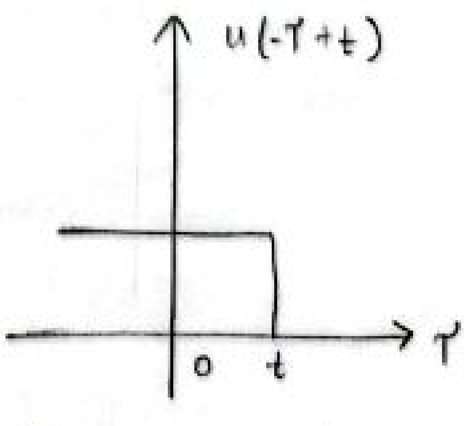
$$s(t) = h(t) * u(t)$$

$$= \int_{-\infty}^{\infty} h(\tau) \cdot u(t-\tau) \cdot d\tau$$









$$s(t) = \int_{-\infty}^{t} h(\tau) \cdot d\tau$$

step response of system

Impulse response of system

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Frequency response of the system :-

method 1

$$\underset{\text{i/p}}{\xrightarrow{x(t)}} \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$$

LTI sys o/p

$$y(t) = x(t) * h(t)$$

$$FT[y(t)] = FT[x(t) * h(t)]$$

$$FT[y(t)] = FT[x(t)] \cdot FT[h(t)]$$

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

method 2

$$\underset{\text{i/p}}{\xrightarrow{X(\omega)}} \boxed{H(\omega)} \longrightarrow Y(\omega) = X(\omega) \cdot H(\omega)$$

LTI sys o/p

$$y(t) = IFT[Y(\omega)]$$

$$y(t) = IFT[X(\omega) \cdot H(\omega)]$$

- $H(\omega) = FT[h(t)] = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} \, dt$

$$h(t) = IFT[H(\omega)]$$

IMP

- $h(t), H(\omega)$ forms F.T pair

$$h(t) \xleftrightarrow{FT} H(\omega)$$

$h(t)$ → Impulse response of the system

$H(\omega)$ → Frequency response of the system

$$\therefore \boxed{H(\omega) = \frac{Y(\omega)}{X(\omega)}}$$

$$H(\omega) = FT[h(t)] = \int_{-\infty}^{\infty} h(t) \cdot e^{-j\omega t} \, dt$$

$H(\omega)$ is always complex

$$H(\omega) = |H(\omega)| \, e^{j\theta(\omega)}$$

where $\theta(\omega) = \angle H(\omega)$

Frequency Spectrum

⇒ The plot b/w $|H(\omega)|$ vs ω is called magnitude spectrum. It posses even symmetry (y-axis)

⇒ The plot b/w $\theta(\omega)$ vs ω is called phase spectrum. It posses odd symmetry (origin)

⇒ the combination magnitude & phase spectrum is frequency spectrum.

Q, The impulse response of the system $h(t) = e^{-3t} u(t)$

Find the frequency response of the system

$$H(\omega) = FT[h(t)]$$

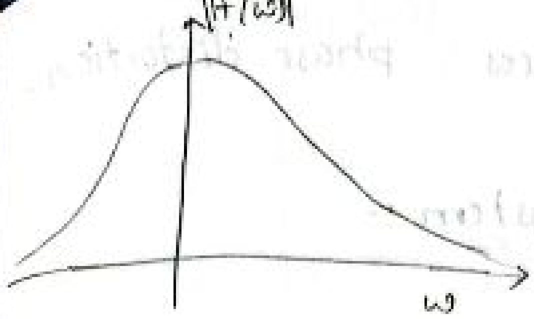
$$= FT[e^{-3t} u(t)]$$

$$= \frac{1}{3+j\omega} \rightarrow \text{freq. response}$$

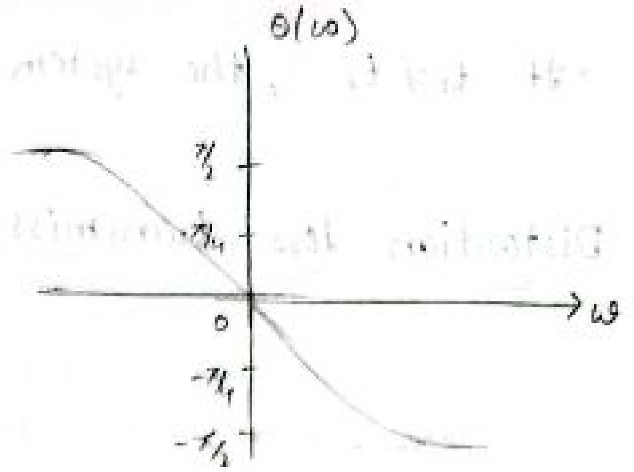
$$\frac{z_1}{z_2} = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

$$|H(\omega)| = \frac{1}{\sqrt{9+\omega^2}} \rightarrow \text{magnitude spec}$$

$$\theta(\omega) = \angle H(\omega) = -\tan^{-1}\left(\frac{\omega}{3}\right) \rightarrow \text{phase}$$



Magnitude



Phase

Q, The frequency response of the system $H(\omega) = \frac{1}{1+\omega^2}$

find the impulse response of the system.

Amplitude distortion (Magnitude distortion) :

$$x(t) = \cos(\omega_1 t) + \cos(\omega_2 t) \xrightarrow{i/p} \boxed{h(t)} \xrightarrow{o/p} y(t) = K_1 \cos(\omega_1 t) + K_2 \cos(\omega_2 t)$$

- If $K_1 \neq K_2$, the system produces amplitude distortion

Frequency distortion [Phase or delayed distortion]:

$$x(t) = \cos(\omega_1 t) + \cos(\omega_2 t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \cos[\omega_1 (t - t_1)] + \cos[\omega_2 (t - t_2)]$$

• If $t_1 \neq t_2$, the system produces phase distortion.

Distortion less transmission system :-

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = K_1 \cos[\omega_1(t-t_1)] + K_2 \cos[\omega_2(t-t_2)]$$

$= \cos(\omega_1 t) + \cos(\omega_2 t)$

$\omega_1 \neq \omega_2$

• If the system is free from Amplitude & phase distortion then it is distortion less transmission system.

• If $K_1 = K_2 = K$, $t_1 = t_2 = t_0$

then $\boxed{y(t) = K\cos[\omega_1(t-t_0)] + K\cos[\omega_2(t-t_0)]}$

$$\Longrightarrow \quad y(t) = K x(t-t_0)$$

Apply FT on b.s

$$Y(\omega) = FT[Kx(t-t_0)]$$

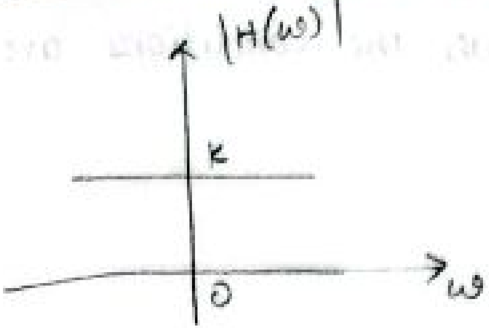
$$= K X(\omega) e^{-j\omega t_0}$$

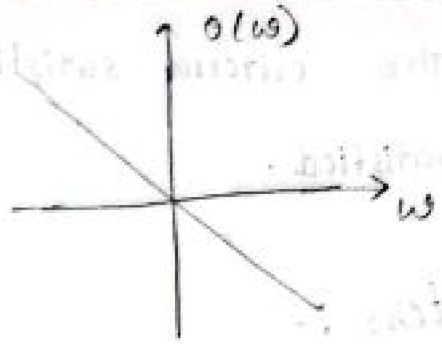
$$\boxed{H(\omega) = \frac{Y(\omega)}{X(\omega)} = K \cdot e^{-j\omega t_0}}$$

$$H(\omega) = |H(\omega)| e^{j\theta(\omega)} = K e^{-j\omega t_0}$$

$$|H(\omega)| = K$$

$$\theta(\omega) = \angle H(\omega) = -\omega t_0$$





- For D.L.T.S ⇒ Magnitude spectrum is constant

  Phase spectrum is linear

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Paley-wiener criteria :-

~~Method~~

- This criteria is used to check whether the system is physically realizable or not

Method-1

- If the system causal & stable then the system is said to be physically realizable

1. $h(t) = 0$ for $t < 0$ – causal system where $h(t)$ is impulse response of sys
2. $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$ – stable system

- Both are physically realizable

Method-2

1. $\int_{-\infty}^{\infty} |H(\omega)|^2\, d\omega < \infty$ where $H(\omega)$ is freq. response of sys
2. $\int_{-\infty}^{\infty} \frac{|\ln|H(\omega)||}{1+\omega^2}\, d\omega < \infty$

Both are physically Realizable

- This criteria satisfies if both the conditions are satisfied.

Filters :-

- Filters allows some freq components, altenuate (remove) some other freq. components

$$x(t) \xrightarrow{\text{i/p}} \boxed{H(\omega)} \xrightarrow{\text{o/p}} 1,\ y(t) = x(t) * h(t)$$

$X(\omega)$ Filter   2, $Y(\omega) = X(\omega)H(\omega)$

$$y(t) = IFT[Y(\omega)]$$

$$= IFT[X(\omega)\, H(\omega)]$$

Types of filters:

1, **Ideal low-pass filters [ILPF] :-**

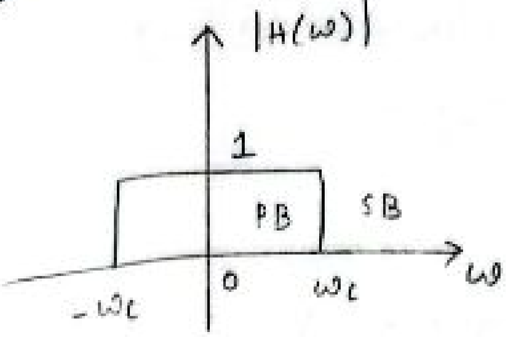
- It allows low freq. components and removes high freq. components
- The filter is characterized by its freq. response

$$H(\omega) = |H(\omega)|\, e^{j\theta(\omega)}$$

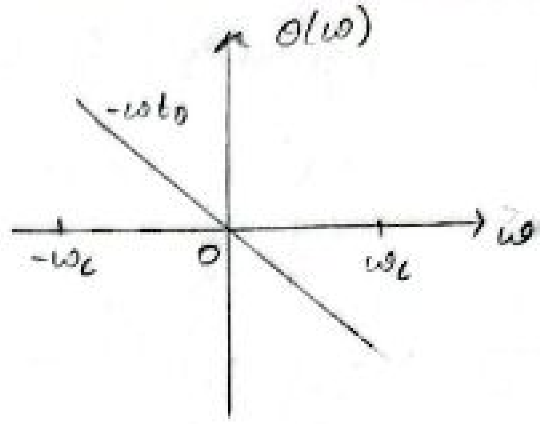
$$\text{where } \theta(\omega) = \angle H(\omega)$$

Magnitude function :- $|H(\omega)| = \begin{cases} 1 & \text{for } -\omega_c \le \omega \le \omega_c \\ 0 & \text{otherwise} \end{cases}$

Phase function :- $\theta(\omega) = \begin{cases} -\omega t_0 & \text{for } -\omega_c \le \omega \le \omega_c \\ 0 & \text{otherwise} \end{cases}$



M.S



P.S

PB - Pause Band

SB - stop Band

ω_c - cut off freq

Freq. response $\Rightarrow H(\omega) = \text{rect}\left(\frac{\omega}{2\omega_c}\right) e^{-j\omega t_0}$

$H(\omega) = |H(\omega)| \cdot e^{j\omega}$

~~imup~~ impulse response $\Rightarrow h(t) = IFT[H(\omega)]$

- $sa(Bt) \xleftrightarrow{FT} \frac{\pi}{B}\text{rect}\left(\frac{\omega}{2B}\right)$

let $B = \omega_c$

$sa(\omega_c t) \xleftrightarrow{FT} \frac{\pi}{\omega_c}\text{rect}\left(\frac{\omega}{2\omega_c}\right)$

$\frac{\omega_c}{\pi} sa(\omega_c t) \xleftrightarrow{FT} \text{rect}\left(\frac{\omega}{2\omega_c}\right)$

- $z(t) = \frac{\omega_c}{\pi} sa(\omega_c t) \xleftrightarrow{FT} Z(\omega) = \text{rect}\left(\frac{\omega}{2\omega_c}\right)$

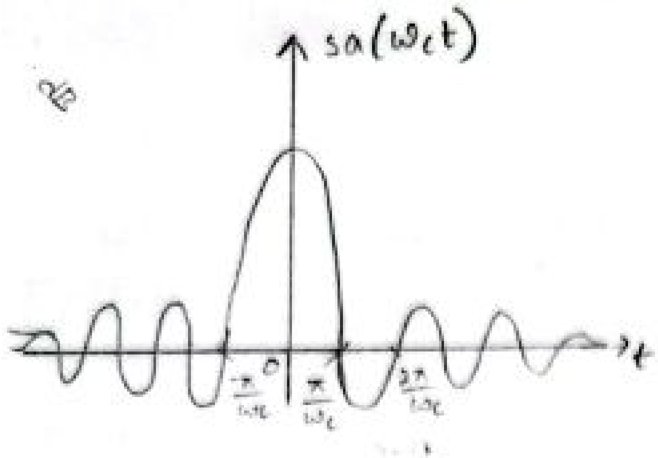
small

$z(t-t_0) \xleftrightarrow{FT} Z(\omega)e^{-j\omega t_0}$

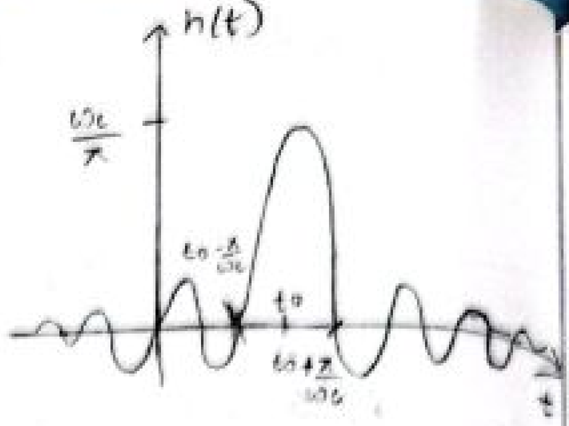
$h(t) = \frac{\omega_c}{\pi} sa[\omega_c(t-t_0)] \xleftrightarrow{FT} H(\omega) = \text{rect}\left(\frac{\omega}{2\omega_c}\right) e^{-j\omega t_0}$

Impulse response of ILPF :-

$$\boxed{h(t) = \frac{\omega_c}{\pi} sa[\omega_c(t-t_0)]}$$



$\Longrightarrow$



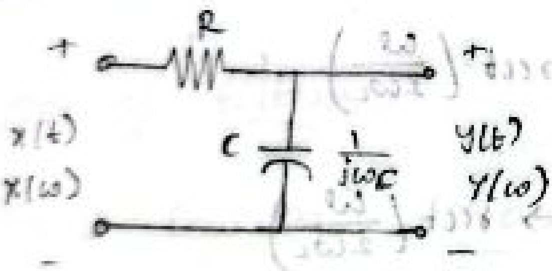
$h(t) \neq 0$ for $t<0$

As $h(t) \neq 0$ for $t<0$ the filter is non-causal & not physically realizable

2> practical LPF :-

RC LPF

for practical we define 3 dB band width



for ideal → 2 → (high, low)

$$Y(\omega) = \left(\frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}\right) X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{1 + j\omega RC}$$

$$H(\omega) = \frac{1/RC}{\frac{1}{RC} + j\omega}$$

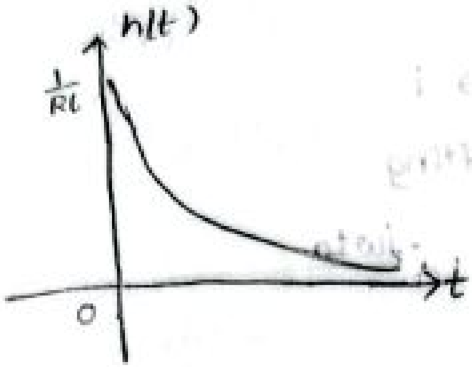
let $a = \frac{1}{RC}$

$$H(\omega) = \frac{a}{a + j\omega} \Rightarrow \text{Freq Response}$$

Impulse Response $\Rightarrow$ $h(t) = IFT[H(\omega)]$

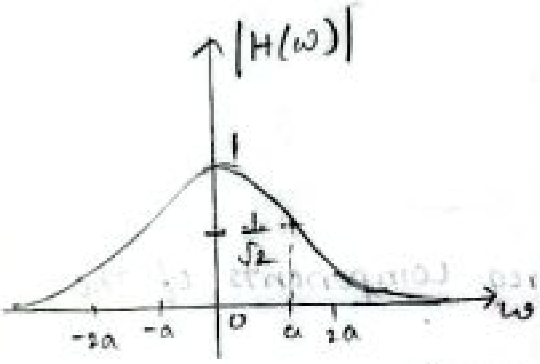
$$h(t) = a e^{-at} u(t)$$

$$h(t) = \frac{1}{RC} e^{-t/RC} u(t)$$



1) $h(t) = 0$ for $t < 0$ causal

2) $\int_{-\infty}^{\infty} |h(t)| dt < \infty$ stable



at $\omega = a$ $\Rightarrow$ its value is $\frac{1}{\sqrt{2}}$

3dB Band width :

The freq at which the mag value of the filter falls down to $\frac{1}{\sqrt{2}}$ time of its max value

$$H(\omega) = \frac{a}{a + j\omega}$$

$$|H(\omega)| = \frac{a}{\sqrt{a^2 + \omega^2}} \quad ; \quad \theta(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

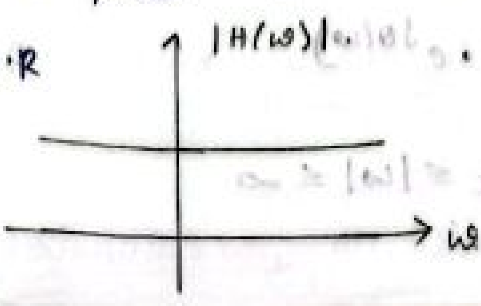
17/10

3) All pass filter

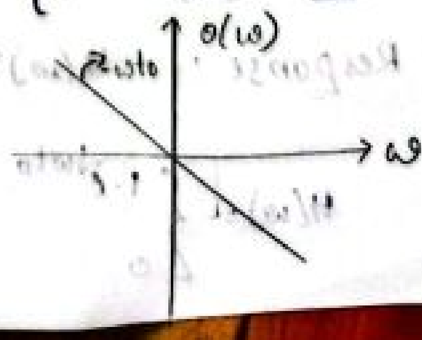
All pass filter passes all freq components

- The Mag. response is 1 $\forall\ \omega$ (i.e $|H(\omega)| = 1\ \forall\ \omega$)
- The phase " is linear (i.e $\theta(\omega) = \angle H(\omega) = -\omega t_0$

M.R



P.R



Frequency Response :- $H(\omega) = |H(\omega)| e^{j\theta\omega}$

$H(\omega) = 1 \cdot e^{-j\omega t_0}$

linear con filter -> find I_m

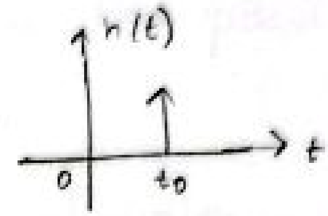
Impulse Response :- $h(t) = IFT[H(\omega)]$

$$\delta(t) \xleftrightarrow{FT} 1$$

time shifting

$$\delta(t-t_0) \xleftrightarrow{FT} 1e^{-j\omega t_0}$$

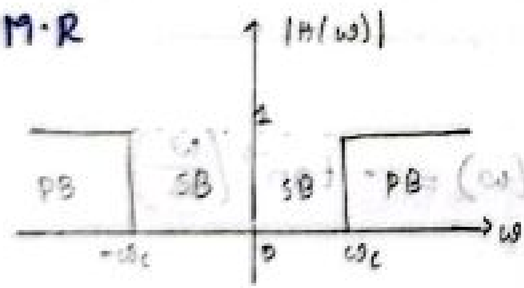
$$h(t) = \delta(t-t_0)$$



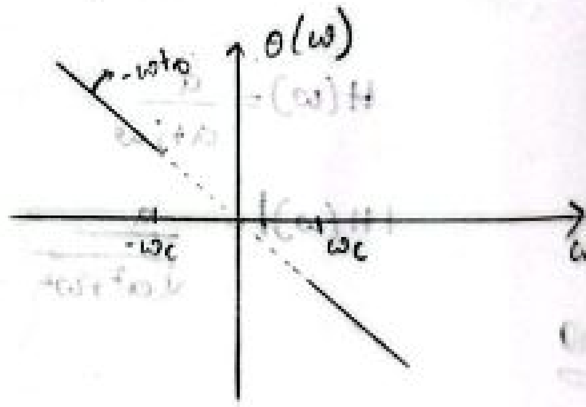
4, Ideal High pass filter

- This filter allows only High freq components of the i/p and removes low freq components.

M·R



P·R



ω_c = cutoff freq

3dB· Band width = a :-

- the freq at which the mag value of the filter falls down to $\frac{1}{\sqrt{2}}$ times of its max value.

Frequency Response :- $H(\omega) = |H(\omega)| \cdot e^{j\theta(\omega)}$

$$H(\omega) = \begin{cases} 1 \cdot e^{-j\omega t_0} & \text{for } \omega_c \le |\omega| \le \infty \\ 0 & \text{else} \end{cases}$$

Impulse Response :-

$$H_{LP}(\omega) + H_{HP}(\omega) = H_{AP}(\omega)$$

$$H_{HP}(\omega) = H_{AP}(\omega) - H_{LP}(\omega)$$

$$IFT[H_{HP}(\omega)] = IFT[H_{AP}(\omega)] - IFT[H_{LP}(\omega)]$$

$$h_{HP}(t) = h_{AP}(t) - h_{LP}(t)$$

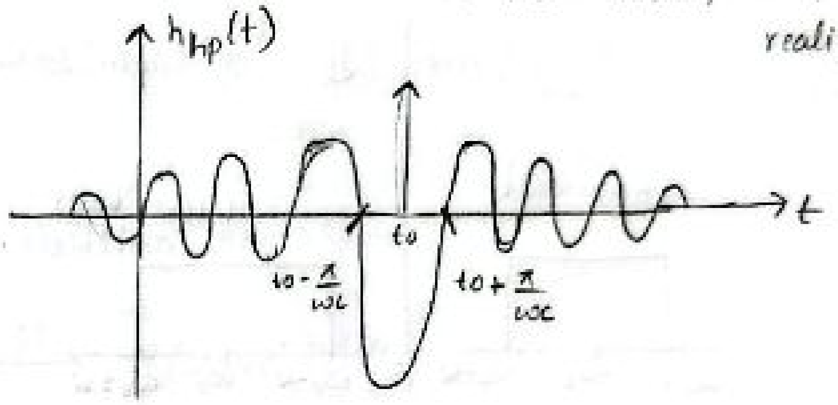
$$\boxed{h_{HP}(t) = \delta(t-t_0) - \frac{\omega_c}{\pi} sa[\omega_c(t-t_0)]}$$

I.R of Ideal HPF

$h_{HP}(t) \neq 0$ for $t < 0$

∴ Non-causal, not physically realizable

I.R ⟹

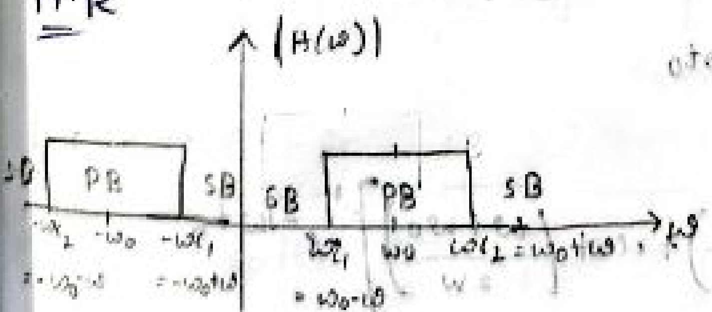


IMP 10M

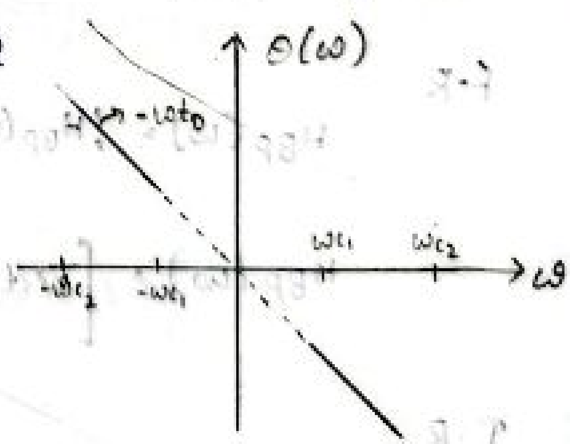
5) Ideal Band pass filter :-

Band pass filter allows the Band of freq of i/p signal & removes all other frequencies of i/p signal

M.R



P.R



ω_0 = center freq

ω_{c_1} = lower cut off freq

ω_{c_2} = upper " " "

Bandwidth $= \omega_{c_2} - \omega_{c_1} = 2W$

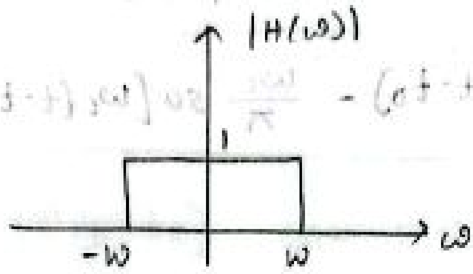
W - const

Frequency Response :- $H(\omega) = |H(\omega)|\, e^{j\theta(\omega)}$

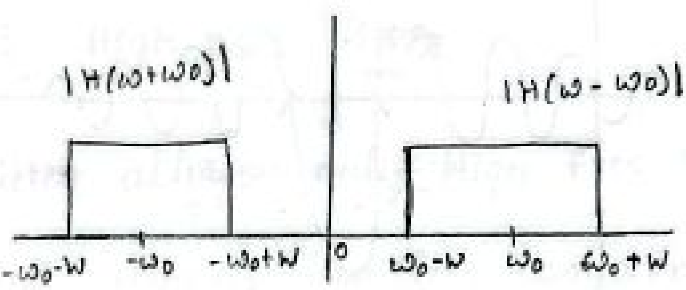
$$H(\omega) = \begin{cases} 1 \cdot e^{-j\omega t_0} & \text{for } \omega c_1 \le |\omega| \le \omega c_2 \\ 0 & \text{else} \end{cases}$$

Impulse Response :-

consider rect pulse



⇓ By right & left shifting



$$|H_{BP}(\omega)| = |H(\omega-\omega_0)| + |H(\omega+\omega_0)|$$

$$|H_{BP}(\omega)| = \text{rect}\left(\frac{\omega-\omega_0}{2W}\right) + \text{rect}\left(\frac{\omega+\omega_0}{2W}\right)$$

F.R

$$H_{BP}(\omega) = |H_{BP}(\omega)|\, e^{-j\omega t_0}$$

$$H_{BP}(\omega) = \left[\text{rect}\left(\frac{\omega-\omega_0}{2W}\right) + \text{rect}\left(\frac{\omega+\omega_0}{2W}\right)\right] \cdot e^{-j\omega t_0}$$

I.R

$$h_{BP}(t) = \text{IFT}\left[H_{BP}(\omega)\right]$$

$$Sa(Bt) \xleftrightarrow{FT} \frac{\pi}{B}\, \text{rect}\left(\frac{\omega}{2B}\right)$$

let $B = W$

$$sa(Wt) \xleftrightarrow{FT} \frac{\pi}{W} \cdot rect\left(\frac{\omega}{2W}\right)$$

Amplitude scaling by a factor $\frac{W}{\pi}$.

$$x(t) = \frac{W}{\pi} \cdot sa(Wt) \xleftrightarrow{FT} X(\omega) = rect\left(\frac{\omega}{2W}\right)$$

modulation theorem

$$x(t) \cdot \cos(\omega_0 t) \xleftrightarrow{FT} \frac{1}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right]$$

$$y(t) = \frac{W}{\pi} sa(Wt) \cos(\omega_0 t) \xleftrightarrow{FT} \underset{H(\omega)=}{} \frac{1}{2}\left[rect\left(\frac{\omega-\omega_0}{2W}\right) + rect\left(\frac{\omega+\omega_0}{2W}\right)\right]$$

time shifting operation

$$y(t-t_0) \xleftrightarrow{FT} Y(\omega) \cdot e^{-j\omega t_0}$$

$$\frac{W}{\pi} sa[W(t-t_0)] \cos[\omega_0(t-t_0)] \xleftrightarrow{FT} \frac{1}{2}\left[rect\left(\frac{\omega-\omega_0}{2W}\right) + rect\left(\frac{\omega+\omega_0}{2W}\right)\right] e^{-j\omega t_0}$$

Amplitude scaling

$$\underset{h_{BP}(t)}{} 2\frac{W}{\pi} sa[W(t-t_0)] \cos[\omega_0(t-t_0)] \xleftrightarrow{FT} \underset{H_{BP}(\omega)=}{} \left[rect\left(\frac{\omega-\omega_0}{2W}\right) + rect\left(\frac{\omega+\omega_0}{2W}\right)\right] e^{-j\omega t_0}$$

I.R of IBPF :-

$$\boxed{h_{BP}(t) = \frac{2W}{\pi} sa\left[W(t-t_0)\right] \cos\left[\omega_0(t-t_0)\right]}$$